Physics PhD candidacy examinations

1984-1985

University of Chicago

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1984-09 ____ 2

1985-09 ____ 14

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DEPARTMENT OF PHYSICS
PH.D. CANDIDACY EXAMINATION
Autumn 1984

Work all six problems. Show your work clearly. Credit will be given only if the work is shown explicitly. Give numerical answers when requested. Use clear, well labeled diagrams wherever appropriate.

1. In order to test a new theory you are required to subject samples to various extreme environmental tests. Your samples will be approximately 1cm x 1cm x 1cm in size. Estimate the following quantities which represent the experimental limits of your laboratory tests.

1. The maximum D.C. magnetic field
2. The maximum D.C. electrostatic field
3. The best vacuum (at room temperature)
4. The maximum static pressure
5. The lowest possible temperature
6. The highest possible temperature
7. The maximum number of slow neutrons/sec
8. The maximum number of fast protons/sec (say at 100 GeV)
9. The maximum number of x-rays/sec (say Cu, $K\alpha$)
10. The maximum number of visible photons/sec

Explain each estimate in a few sentences.

2. Consider a particle of mass m in a box. The box is represented by the boundary conditions:

$$\psi(x_1+\ell_1,x_2,x_3) = \psi(x_1,x_2+\ell_2,x_3) = \psi(x_1,x_2,x_3+\ell_3) = \psi(x_1,x_2,x_3)$$

(a) Show that the Schrödinger equation admits a general solution

$$\psi = \exp i\,(\vec{k}.\vec{x} - \omega t)$$

(b) Give the allowed values of $\vec{k}$.

(c) Show that the kinetic energy is quantized.

(d) What is the number of states in an energy interval dE?

(e) Consider N particles in the box, without interaction, obeying Fermi statistics.

--What is the configuration which minimizes the total kinetic energy?

--What is the Fermi energy?

--Calculate it for Lead in Electron Volts (mass number 207, density 11.6×10^3 kg m^{-3}, two electrons per atom).

--Compare the Fermi velocity with room temperature thermal velocity.

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3. A cylindrical laser beam, of cross section s, frequency ν and power P, is directed vertically upwards onto a spherical particle whose diameter 2r is much smaller than the beam. The particle is levitated, its weight being balanced by the resulting force of its interaction with the laser beam.

(a) Supposing that the particle totally absorbs light, derive an expression for the force produced by the beam on the particle.

(b) What is the beam power needed to obtain levitation?

Apply your result to the following application:

density = $\rho = 2.5 \times 10^3$ kg m^{-3}

$r = 10\ \mu$m

$s = 0.2$ mm^2

$g_o = 9.8$ ms^{-2}

(c) Suppose that the levitating particle totally reflects light. What is the laser power needed to obtain levitation?

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4. Determine the moment of inertia and the intermolecular distance of the $^1H^{35}Cl$ molecule, if the difference $\Delta\bar{\nu}$ in the wave number of two neighboring lines of the rotational (infrared) band of HCl is equal to 20.9/cm.

Evaluate the corresponding $\Delta\bar{\nu}$ for the $^2D^{35}Cl$ molecule. (D stands for deuterium.)

5. Calculate the effective scattering cross section σ for a small highly conducting sphere of radius a exposed to electromagnetic radiation of frequency ω, assuming that $a\omega/c \ll 1$. Under these circumstances, how much more strongly is blue light ($\lambda = 4{,}000$ Å) scattered than red light ($\lambda = 7{,}000$ Å)?

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6. Consider a particle bound in an arbitrary one-dimensional potential well. Let $U_a(x)$ and $U_b(x)$ be normalized eigenfunctions of the Hamiltonian corresponding to non-degenerate eigenvalues E_a and E_b respectively.

Suppose that at $t = 0$, the particle is in a state which is an arbitrary linear combination of $U_a(x)$ and $U_b(x)$.

(a) Find $\langle E \rangle$. Is it time dependent?

(b) Find $\langle p \rangle$. Under what conditions will it be time dependent?

(c) For the case of an harmonic oscillator, what are the conditions on E_a and E_b for $\langle p \rangle$ to be time independent?

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7. A method used at SLAC to produce energetic photons is to back scatter laser photons from the high energy electron beam. Given that the electron beam energy = 20 GeV and that the laser beam energy = 2.4 eV what is the back scattered photon energy? At what angle θ from the beam direction does the photon energy drop to half its peak value?

8. The interaction energy between two identical atoms of spin angular momentum S is:

$$U_{ik} = -2J\,\vec{S}_i \cdot \vec{S}_k \qquad (1)$$

where J is a positive constant.

(a) What is the physical meaning of J? Do you know of an interaction where J is negative?

(b) Show that (1) can be thought of as the effect of a magnetic field

$$\vec{B} = \frac{-2J}{g\mu_B}\,\vec{S}_i \text{ on atom k}$$

where μ_B is the Bohr magneton, g a gyromagnetic ratio.

(c) Consider a one-dimensional linear array of atoms, of equal separation a where the interactions are nearest neighbor only. What is the torque on atom i from atoms i-1 and i+1?

(d) Write down the differential equation relating S_{i-1}, S_i, S_{i+1} and dS_i/dt.

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9. The small amplitude equation for a forced undamped pendulum is:

$$\ddot{x} + \alpha x = F \cos \omega t \qquad (1)$$

and we will concentrate on periodic solutions only.

(a) Show that the solution of (1) contains a term $Ct \sin \omega t$, for the resonance condition $\omega = \sqrt{\alpha}$. What is the problem with this so-called secular term?

(b) For larger amplitude oscillation the pendulum equation can be written, (β small)

$$\ddot{x} + \alpha x + \beta x^3 = F_0 \beta \cos \omega t \qquad (2)$$

Close to resonance one can rewrite this as

$$\ddot{x} + \omega^2 x = (\omega^2 - \alpha)x - \beta x^3 + F_0 \beta \cos \omega t \qquad (3)$$

To solve (3) by iteration, insert the zero order solution $x_0 = A \cos \omega t$ with $\omega = \sqrt{\alpha}$ into the right hand side of (3). It leads to the next approximation

$$x_1 + \omega^2 x_1 = P_1 \cos \omega t - 1/4\, \beta A^3 \cos 3\omega t \qquad (4)$$

(Use the identity $\cos^3 \omega t = 3/4 \cos \omega t + 1/4 \cos 3\omega t$)

Calculate P_1.

(c) To avoid secular terms in (4) what should be the value of P_1 ?

(d) Deduce the relation between amplitude and frequency for this P_1 and write down the solution for x_1.

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10. A particle (mass m) is confined to move on a circle (radius a) under the influence of a potential

$$V(\theta) = A \cos(3\theta)\ , \ (\theta \text{ in radians})$$

(a) Write down the Hamiltonian H and the Schrödinger equation for this system.

(b) Find symmetry operators which commute with H, and determine their eigenvalues.

(c) Determine approximate eigenvalues and eigenfunctions of the three lowest states for the two cases $ma^2A/\hbar^2 \ll 1$ and $ma^2A/\hbar^2 \gg 1$. (Do not attempt to derive rigorous solutions.)

11. In a nuclear explosion a U^{238} nucleus is subject to an intense pulse of neutrons. Give a numerical value for the probability that it will absorb 20 neutrons in succession if the absorption cross section for all nuclei^{238} up to U^{257} is the same and the neutron pulse is short compared to any decay time in nuclei in the U^{238} to U^{257} chain.

Assume the following:

neutron flux $F = 10^{33}$ neutrons/cm^2 sec
neutron pulse length $\tau = 10^{-8}$ sec
neutron absorption cross section $\sigma = 6 \times 10^{-24}$ cm^2

If the efficiency for collecting post explosion debris is 10^{-9}, apart from decay, how many U^{258} atoms will be in the collected sample per kg of U^{238} in the original device?

12. A cylindrical cavity of radius 2cm and height 10cm is cut inside an infinite block of magnetic material. A solenoid made with copper wire has an outer radius of 2cm, an inner radius 1cm and height 10cm and is inserted into the cavity. Given that the permeability of the magnetic material is 10^3 calculate the power that must be dissipated in the coil to produce a magnetic field of 750 gauss at the center of the cavity. (Assume that the solenoid is completely filled with conductor and that copper has a resistivity of 2×10^{-6} ohm cm.)

Also calculate the inductance of the coil as a function of the number of turns in the solenoid (N).

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13. Multiple Choice (work need not be shown)

(a) The reflectivity (reflected intensity/incident intensity) of ordinary glass for visible light at normal incidence is about

0.001 0.01 0.05 0.1 0.2

(b) For electron elastic scattering by a ^{4}He atom, what is the correct general statement about the polarization of the scattered particle when the incident electron is not polarized?

(i) Not polarized

(ii) Polarized:

in the incident direction

in the scattered direction

perpendicular to plane of scattering

(c) Indicate which of the decay modes can actually occur:

$K^+ \to \pi^+ + \pi^0$

$e^+ + \nu_e$

$\pi^- + \mu^+ + \mu^+$

$\pi^+ + \gamma$

(d) and (e) on following page . . . .

13. (continued)

(d) For the following systems, the spin J (total angular momentum) and parity P of the ground state are:

	J	P
hydrogen atom		
helium 3 nucleus		
carbon 12 nucleus		
positronium		

(e) The ionization energy loss of a particle (mass m, charge Ze, velocity v) traversing a unit length of matter is (up to a Logarithmic factor) proportional to

(i) $\frac{Z}{m}$ (ii) $\frac{Z^2}{v}$ (iii) $\frac{Z^2}{v^2}$ (iv) $\frac{Z}{v^2}$

14. If a ball is dropped from a position (x_o, y_o) onto a curved surface, it will start bouncing. Assume perfect elasticity. Let the profile of the surface be a parabola

$$y = ax^2/2, \quad \text{(x:horizontal, y:vertical, a:positive)}$$

and the motion be confined to the x-y plane.

Assuming that the initial position is infinitesimally close to the axis of the parabola, study the stability of the bouncing motion, that is to say, whether or not the ball will always stay close to the axis, and if it does, under what conditions.

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15. Two electrons, in fixed positions on the z-axis, have a magnetic dipole-dipole interaction (energy)

$$E = C[\vec{s}_1 \cdot \vec{s}_2 - 3s_{1z}s_{2z}]$$

$(2\vec{s}_1 = \vec{\sigma}_1 = \text{Pauli spin matrix}, C = \text{constant})$

(a) Express E/C in terms of the total spin operator

$$\vec{s} = \vec{s}_1 + \vec{s}_2$$

(b) State the eigenvalues of E/C and their degrees of degeneracy (statistical weights g).

(c) Write the partition function and the free energy of the system at absolute temperature T (ignore all but spin degrees of freedom).

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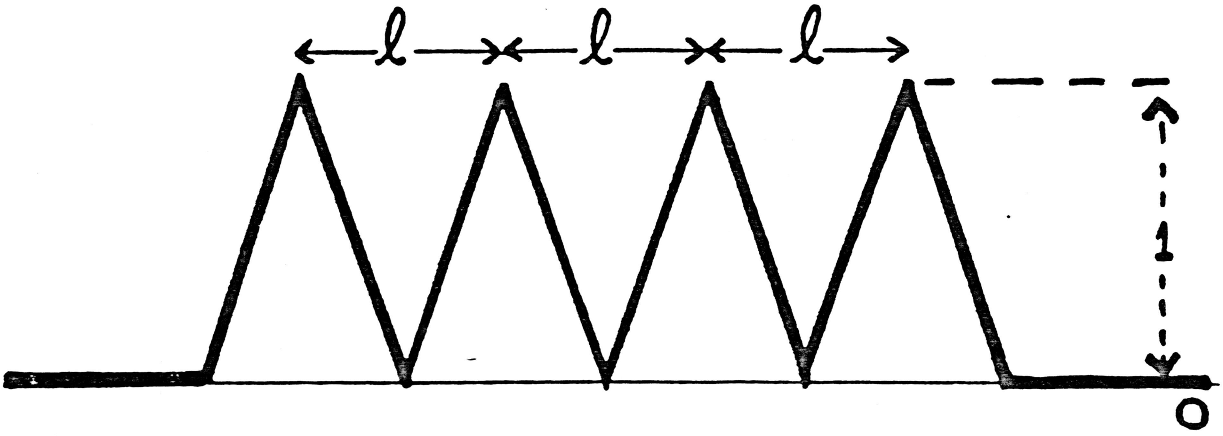
16. What is the ratio of the cross sections for the two reactions

$$\pi^+ + d \rightarrow p + p$$

$$p + p \rightarrow \pi^+ + d$$

in terms of the (same) center of mass energy W?

17. At a particular wavelength λ the amplitude transmission function of a 4-toothed diffraction grating is that shown below.



Knowing that the diffraction pattern produced by this grating is the squared modulus of the Fourier transform of this function--you should be able to sketch that pattern. Indicate as many salient points as you can, e.g., maxima, minima, zeroes, relative intensities, etc.

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18. In fluid flow the main equation is the Navier-Stokes equation:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u} = -\frac{1}{\rho}\vec{\nabla}P + \nu\Delta\vec{u} \qquad (1)$$

(∇ is the gradient and Δ is the Laplacian)

where $\vec{u}$ is the fluid velocity, ρ the density, P the pressure and ν the kinematic viscosity.

(a) What is the dimension of the kinematic viscosity? What is its relation to the ordinary viscosity?

(b) The Reynolds number R is the ratio of the inertial term $\vec{u}\cdot\nabla\vec{u}$ and the dissipative term $\nu\Delta\vec{u}$. What is the dimension of R?

(c) Estimate and compare the R numbers for a 100m runner ($V = 10\text{m s}^{-1}$) and 100m swimmer ($V = 1\text{ms}^{-1}$).

Air viscosity 18×10^{-6} kg $\text{m}^{-1}\text{s}^{-1}$ density 1.2 kg m^{-3}

Water viscosity 10^{-3} kg $\text{m}^{-1}\text{s}^{-1}$ density 10^{3} kg m^{-3}

Take 1m for the size of the athlete. What do you conclude from these results?

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1. Two atoms, each of mass m, interact with each other via a central potential (called a Lennard-Jones potential)

$$U(r) = U_o \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

where r is the distance separating the centers of the atoms and U_o and σ are constants. Initially the atoms are at rest with respect to one another at the equilibrium distance of separation r_o. At some time t_o an external force is applied which acts only in the radial direction and increases the radial separation of the particles to a distance $r = r_o + x$ where $x/r_o \ll 1$. The external force is applied for a short period of time and then turned off. The two atoms will then oscillate about $r = r_o$.

a) Find the equilibrium position r_o.

b) Find the frequency of oscillation in terms of m, U_o, and σ .

c) For argon atoms $U_o = 42 \times 10^{16}$ ergs, $m = 6.6 \times 10^{-23}$ grams and $\sigma = 3.4$ Å. Evaluate the period of oscillation.

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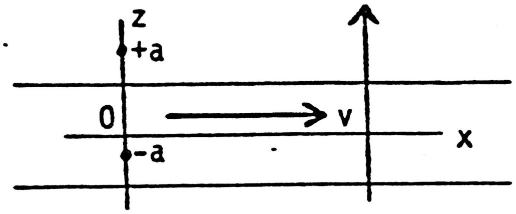
2. Consider the atomic nucleus to be a noninteracting gas of Z protons and of N neutrons, in a common potential well of depth B. What is the energy of the system in its lowest energy state, and for what fraction of protons, $y = Z/(Z + N)$, is the system most bound? Assume the nucleus has a fixed density.

3. Blackbody radiation is in thermal equilibrium with a collection of nucleons which have a matter density ρ. The equations of state of the system are $P = \frac{a}{3} T^4$ and $E = a T^4 V$ where P is the pressure, E the energy, V the volume, T the temperature and a is a constant. The sound speed is defined as $\lambda = \sqrt{(\partial P/\partial \rho)|_{S,N}}$ where S is the entropy and N the number of nucleons. Evaluate the entropy and the sound speed, and see if the expression for λ can ever exceed the speed of light. What do you think of the result?

4. It is desired to determine the lifetime of an unstable particle. Given N measurements of the (proper) time between creation and disintegration of this particle, $(t_1, t_2, \ldots, t_N)$, find an expression for the lifetime and its error.

Suppose now that the detector had no sensitivity to disintegrations between $t = 0$ and $t = T$; find an expression for the particle lifetime, again given N measurements.

5. A conducting liquid flows in the space between two parallel insulating plates, $z=+a$ and $z=-a$. The flow velocity is everywhere along the the ox direction, and its magnitude v is a function of z only. The fluid is incompressible, with viscosity η and electrical conductivity σ. A DC magnetic field B is applied along the z direction.



Let us recall that two fluid parts, separated by a plane $z=z_0$ exert on each other a viscous force equal to $\eta\frac{\partial v}{\partial z}$ per unit surface

a) show by calculating the balance of forces on a volume element in steady state that:

$$\eta\frac{\partial^2 v}{\partial z^2} + f_x - \frac{\partial p}{\partial x} = 0$$

where p is the pressure, and f_x, the volume density of the electromagnetic force, projected along the ox direction. Does the system satisfy the continuity equatio

b) using the electromagnetic equations for a moving conductor in a fixed reference frame, show that:

$$\frac{c^2}{4\pi\sigma}\nabla^2\vec{B} + \nabla\times(\vec{v}\times\vec{B}) = 0$$

and

$$\vec{f} = \frac{1}{4\pi}(\nabla\times\vec{B})\times\vec{B} \quad .$$

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6. Consider a neutral spin 1/2 particle at rest with a permanent electric dipole moment $\vec{p}$ and a magnetic dipole moment $\vec{\mu}$. The particle is in an electric field E in the x direction and, as well, a magnetic field B in the z direction.

Find the energies and eigenstates for the particle.

Show that as $E \to 0$, the energies and eigenstates have the expected limit.

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7. A linearly damped harmonic oscillator is driven with a forcing term $\frac{f}{m} e^{i\gamma t}$. The damping coefficient is λ and natural frequency ω_0. Find the steady state solution, show that by "tuning" the frequency γ that the amplitude shows resonant behavior, and give the height and full width at half-maximum of the resonance.

8. The following may be considered as a non-relativistic model of particle decay.

Consider a system of N particles with arbitrary masses which are held together with compressed springs between them. Let the total energy contained in the (massless) springs be Q. When the constraint holding the particles together is released, the particles will statistically share the energy Q. Let the recoil energy of the i'th particle be T_i.

a) What final state configuration will yield the maximum kinetic energy (max T_i) for the i'th particle?

b) Show that the mean of the maxima is given by

$$<\max T_i> = \frac{N-1}{N} Q$$

where the mean is taken over all the particles.

9. Previously you showed that if the atomic nucleus was considered to be a noninteracting gas of Z protons and N neutrons in a common potential well of depth B, then the total energy had the form

$$E_{tot} = - B A + \frac{\varepsilon_0}{A^{2/3}} (N^{5/3} + Z^{5/3}),$$

where $A = Z + N$ and ε_0 is a constant. Assume the proton charge density to be uniformly distributed. What is the additional energy due to Coulomb repulsion, and how does it affect the value of $y = Z/A$ for the most bound state?

Hint: examine $y \simeq 1/2$. Assume that the nucleus maintains constant density.

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Take the sun as a blackbody with emitted power:

$$d^3P = \frac{2h\nu^3 \cos\theta}{c^3(e^{h\nu/k_BT}-1)} \, dS \, d\Omega \, d\nu$$

where dS is the area element, $d\Omega$ is the solid angle, $d\nu$ is the frequency band, and θ is the angle with respect to the normal surface.

a) Show that the total power emitted from one square meter of the sun is given by:

$$P = \sigma T^4 \qquad \text{(Stefan law)}$$

Calculate σ

(Hint: $\int_0^\infty \frac{y^3 dy}{e^y - 1} = \frac{\pi^4}{15}$)

b) Find the wavelength λ_0 for which the power is a maximum and determine $h/\lambda_0 k_B T$ (Wien Law).

c) Given the solar radius $a = .7\text{x}10^6$ km and the sun-earth distance $D = 1.49\text{x}10^8$ km, calculate E_0 the power received on earth per square meter.

The experimental finding is that $E_0 = 1.35\text{x}10^3$ W/m^2. What is the value of the solar temperature? Is it in agreement with Wien's law?

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11. Show that no electric charge can be held in equilibrium at a point by any arrangement of static charges unless one of them is at the point. What is responsible for the stability of the ionic crystal NaCl? Give a rough estimate of the energy of dissociation of 1 gm of NaCl. (The NaCl lattice is cubic with a spacing of 2.81 A).

12.

(a). The Schrödinger equation for a free particle is given by

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) = \varepsilon\,\Psi(\vec{r}) \qquad (1)$$

where m is the mass of the particle.

Assuming plane wave solutions for $\Psi(\vec{r})$ obtain the energy ε as a function of the momentum $\vec{p}$ of the particle.

(b). Assume the particle is in a uniform magnetic field $\vec{B} = \hat{z}B$. The Schrödinger equation satisfied in this case, in the gauge where the vector potential $A_y = A_z = 0$; $A_x = -By$, is

$$\left[\frac{1}{2m}\left(-ih\frac{\partial}{\partial x} + eBy/c\right)^2 - \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\right]\Psi = \varepsilon\Psi\,. \qquad (2)$$

As $B \to 0$ this reduces to (1) above. Physically one expects the motions in the x-y plane and z directions to be uncoupled. Look for a separable solution with plane waves in the x and z directions. You should be able to map the resulting problem for the y-direction onto a problem you know how to solve. Express the energies ε in (2) in terms of the energies of the problem you should know how to solve.

c). Take the limit as $B \to 0$ of your solution in part (b) for ε. Do you recover the result from part (a)? Explain briefly and qualitatively.

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13. Consider a uniform cylinder of radius R (small compared to a or b) rolling without slipping on a curve $1 = (\frac{x}{a})^2 + (\frac{y}{b})^2$. It starts at $y = 0$, and accelerates under the force of gravity g. What is the rotational energy at the lowest point on the curve?

14. The universe is filled with microwave radiation (3^{o} radiation) which is thought to be a remnant of the "big bang". High energy cosmic ray protons from distant sources can scatter from this radiation. Protons with an energy above the threshold for pion production will be severely attenuated; calculate this energy and estimate the probability of an interaction of a proton of this energy per light year of travel through space.

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15. A classical particle is initially known to be located at the origin of a two-dimensional square lattice. This particle is undergoing Brownian motion and we want to estimate how long it will take the particle to move to the "edge" of the crystal. The crystal has a width 2L. The particle hops from site to site due to its interactions with the atoms forming the lattice.

We assume that we can break up time into steps of width Δ, and the time after n such steps is $t_n = t_{n-1} + \Delta$ where n is an integer ($t_n = n\Delta$). We then assume the position of the particle at time t_{n+1} is given by

$$X^i(t_{n+1}) = X^i(t_n) + \Delta V^i(t_n)$$

where i is a vector label (i=x or y). The velocity $V^i(t_n)$ at time t_n is zero on average, $\langle V^i(t_n)\rangle = 0$, and is uncorrelated with the velocity at any other time, or with the velocity in the transverse direction,

$$\langle V^i(t_n)\, V^j(t_{n'})\rangle = \delta_{i,j}\ \delta_{n,n'}\ \frac{D}{\Delta}$$

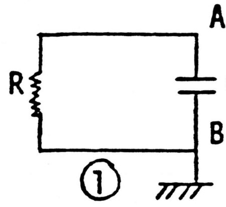
where D is the diffusion coefficient and is clearly associated with the probability that the particle will hop from one lattice site to another in time Δ. We also assume that the velocity of the particle and its position at the same time are uncorrelated

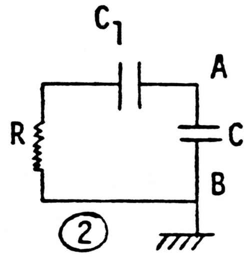
$$\langle X^i(t_n)\, V^j(t_n)\rangle = 0.$$

Compute the average time it will take the particle to reach the edge of the crystal in terms of L and D. The average distance from the origin V at time t_n is given by $\sqrt{\langle \vec{X}^2(t_n)\rangle}$.

16.

a) Consider circuits 1 and 2:





Calculate $\overline{v^2}$ the mean square fluctuating voltage between A and B and the mean energy stored in condenser C, for both circuits.

(Hint: A resistance is equivalent to a source of $\overline{v^2} = 4Rk_BT\,\Delta\nu$ in series with an ideal resistance R where $\Delta\nu$ is the bandwidth.)

b) Consider a dipole AB, consisting of a source $e = E \exp i\omega t$ in series with an internal impedance $Z = r + ix$.

A measuring meter of internal resistance g and of bandwidth $\Delta\nu$ is applied at AB.

For the following types of meters, calculate the signal to noise ratio and the optimum value of g:

i) galvanometer type (current sensitive)
ii) electrometer type (voltage sensitive)

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17. The microscopic form of Maxwell's equations can be written in the form

$$\nabla\cdot\vec{E} = 4\pi\rho \tag{1}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial}{\partial t}\vec{E} \tag{2}$$

$$\nabla\cdot\vec{B} = 0 \tag{3}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \tag{4}$$

where $\vec{E}$ is the electric field, $\vec{B}$ the magnetic field, ρ the charge density and $\vec{J}$ the charge current density. It is often convenient to break ρ and $\vec{J}$ into an external charge and current acting on a system of interest so

$$\rho = \rho_{\text{ext.}} + \rho_I$$

$$\vec{J} = \vec{J}_{\text{ext.}} + \vec{J}_I$$

where I stands for internal. Typically, one can rewrite (1) and (2) in the form

$$\nabla\cdot\vec{D} = 4\pi\rho_{\text{ext.}} \tag{5}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J}_{\text{ext.}} + \frac{1}{c}\frac{\partial}{\partial t}\cdot\vec{D} \tag{6}$$

where the Fourier transform in space and time of the displacement field $\vec{D}$ is related to the total electric field by

$$\vec{D}(\vec{q},\omega) = \varepsilon(\vec{q},\omega)\,\vec{E}(\vec{q},\omega).$$

and ε is the dielectric "constant". It is usual to introduce the wave-number ($\vec{q}$) and frequency (ω) dependent conductivity $\sigma(\vec{q},\omega)$ via the generalization of Ohms law

$$\vec{J}_I(\vec{q},\omega) = \sigma(\vec{q},\omega)\,\vec{E}(\vec{q},\omega).$$

Show, using the results above, how $\varepsilon(\vec{q},\omega)$ and $\sigma(\vec{q},\omega)$ are related. Consider the long wavelength and low frequency behavior of this system. What is the condition for a dielectric material? What is the condition for a conducting material? How must ε and σ behave in these cases?

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18. Consider the ground state of boron (^5B).

a) What degeneracy is lifted with an L·S term in the Hamiltonian?

b) Calculate the effect of the application of a weak magnetic field: what are the quantum numbers of the levels and what are their (relative) energies?

Note that the Landé g value is given by

$$g = 1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j\,(j+1)}$$